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Main Theme

Energy Efficiency, Reality and Perspectives

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A Level-Set Immersed Boundary Method For Simulating Flows Around Cylinders In Tandem And Side-By-Side Arrangements

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Contents

1 Immersed boundary method / Level set method

2 Numerical solver

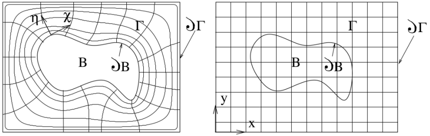
3 Numerical results

4 Conclusion and perspectives

Immersed boundary method / Level set method

Immersed boundary method

Simulate flows on grids that de not conform to the shape of the boundaries

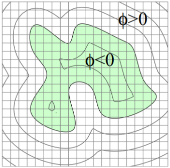


- ➨ Allow practitioners to side-step costly mesh generation process
- ➨ Arbitrary geometry is superimposed over a uniformly Cartesian grid
  - ✔ Apply a boundary closure rule to governing equations
  - ✔ Block-structure allows for simple AMR to resolve structures near boundary
- ➨ Simple, structured data layout with fixed spacing has favorable characteristics for HPC compared with e.g. unstructured approach
- ➨ Challenges
  - ✔ Reduction of order-of-accuracy at boundaries
  - ✔ **Numerical instability at boundaries since it can not capture smoothly the interface**
  - ✔ **Errors to the interpolation of kernels**

Accurate Conservative Level Set Approach

- The interface is defined as zero level set of a distance function $\phi(\mathrm{x}, t)$ from the interface.
  - ✔ The level set scalar is assumed to be a signed distance function to the interface

$$\phi(\mathrm{x}, t) = \begin{cases} -d & \text{inside the immersed solid} \\ 0 & \text{at the fluid-solid interface} \\ +d & \text{inside the fluid} \end{cases}$$



- The conservative level set approach defines the phase interface to be the $\psi(\mathrm{x}, t) = 0.5$ isosurface of

$$\psi(\mathrm{x}, t) = \frac{1}{2}\left(\tanh\left(\frac{\phi(\mathrm{x}, t)}{2\varepsilon}\right) + 1\right),$$

  - ✔ $\psi(\mathrm{x}, t)$ is transported via the material evolution equation

$$\frac{\partial \psi}{\partial t} + \nabla \cdot (\psi \mathbf{u}) = 0.$$

Accurate Conservative Level Set Approach

- Re-initialization is used to restore ψ to a hyperbolic tangent profile after transport (Chiodi and Desjardins, 2017; Sahut et al., 2020)

$$\frac{\partial \psi}{\partial \tau} = \nabla \cdot \left[\frac{1}{4\cosh^2\left(\frac{\phi_{\mathrm{map}}}{2\epsilon(\boldsymbol{x})}\right)} \left(\nabla \phi_{\mathrm{map}} \cdot \mathrm{n}_{\mathrm{FMM}} - \mathrm{n}_{\mathrm{FMM}} \cdot \mathrm{n}_{\mathrm{FMM}}\right) \mathrm{n}_{\mathrm{FMM}} \right]$$

with

- $\phi_{\mathrm{map}}(\boldsymbol{x}, \tau) = \epsilon(\boldsymbol{x}) \log\left(\frac{\psi(\boldsymbol{x},\tau)}{1-\psi(\boldsymbol{x},\tau)}\right)$ is the inverse of the conservative level set function.
- $\mathrm{n}_{\mathrm{FMM}}$ is computed as

$$\mathrm{n}_{\mathrm{FMM}}(\boldsymbol{x}, t) = \frac{\nabla \phi_{\mathrm{FMM}}(\boldsymbol{x}, t)}{\|\nabla \phi_{\mathrm{FMM}}(\boldsymbol{x}, t)\|}$$

  with ϕ_{FMM} a distance function computed from a Fast Marching Method algorithm

- The construction of ϕ_{FMM} removes all oscillatory behaviors of ψ in the computation of normals.

Numerical solver

Numerical solver

In each phase, the material properties are constant, which allows to write $\varrho = \varrho_s$ in the solid immersed body, while $\varrho = \varrho_f$ in the fluid, and $\mu = \mu_s$ in the solid and $\mu = \mu_f$ in the fluid.

$$\frac{\partial \varrho}{\partial t} + \mathfrak{u} \cdot \nabla \varrho = 0$$

$$\frac{\partial \mathfrak{u}}{\partial t} + (\mathfrak{u} \cdot \nabla)\mathfrak{u} = -\frac{1}{\varrho}\nabla \mathfrak{p} + \frac{1}{\varrho}\nabla \cdot \left(\mu \left[\nabla \mathfrak{u} + \nabla \mathfrak{u}^{\intercal}\right]\right),$$

- At the interface, the material properties are subject to a jump that is written $[\varrho]_\Gamma = \varrho_f - \varrho_s$ and $[\mu]_\Gamma = \mu_f - \mu_s$.
- The velocity field is continuous across the interface, $[\mathfrak{u}]_\Gamma = 0$.
- The existence of the pressure jump induces a discontinuity in the pressure at the interface Γ, and one can write

$$[\mathfrak{p}]_\Gamma = \sigma\kappa + [\mu]_\Gamma \, \boldsymbol{n}^{\intercal} \cdot \nabla \mathfrak{u} \cdot \boldsymbol{n}.$$

- The solid–fluid coupling is achieved due to the Ghost-Fluid Method (GFM)

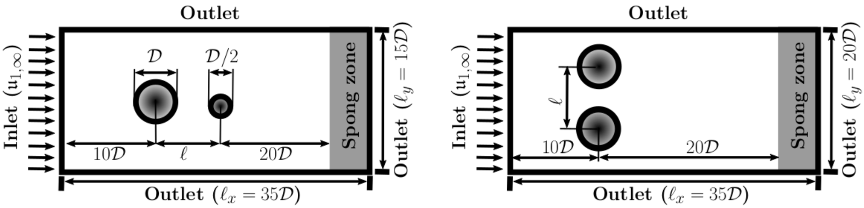
Numerical solver

- Projection method based on fractional time steps developed by Chorin (1968) and improved by Kim and Moin (1985).
  - ✔ Fourth-order central scheme is used for the spatial integration
  - ✔ Third-order accurate semi-implicit Crank-Nicolson scheme is employed for time integration.
- Poison equation is solved using the Livermore's `Hypre` library with the PCG (pre-conditioned conjugate gradient) method.

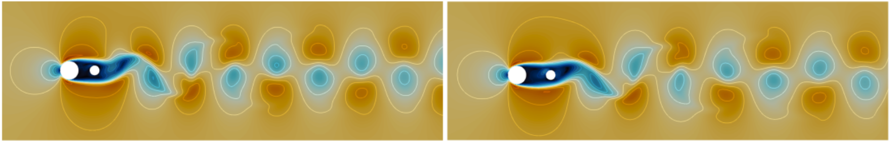
Numerical results

Numerical results

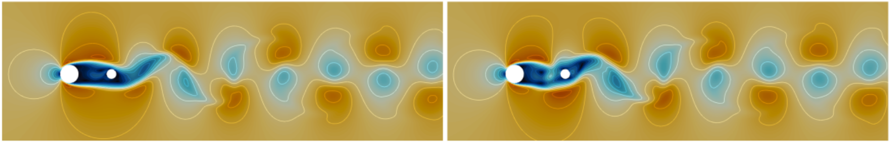
- The downstream cylinder is located at $10\mathcal{D}$ from the inflow boundary, while the upstream cylinder is positioned at $10\mathcal{D}$ from outgoing boundary.
- A sponge layer to damp out the possible reflection of pressure fluctuations at the outlet.
- $\mathcal{R}e = \varrho u_{1,\infty}\mathcal{D}/\mu = 200$
- $1.5\mathcal{D} \leq \ell \leq 4.0\mathcal{D}$
- $\mathcal{N}_x \times \mathcal{N}_y = 800 \times 300$



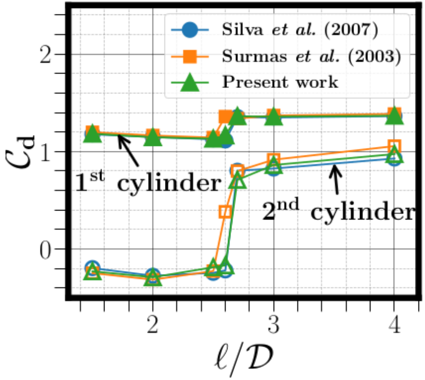
Numerical results

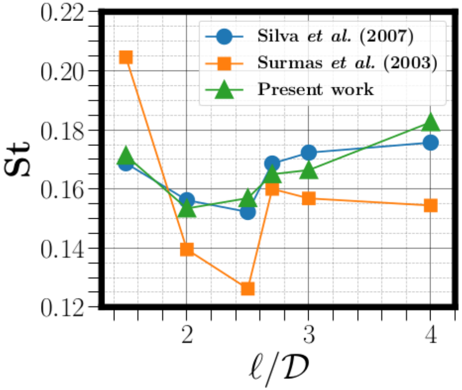


(a) $\ell/\mathcal{D} = 1.5$ (b) $\ell/\mathcal{D} = 2.0$

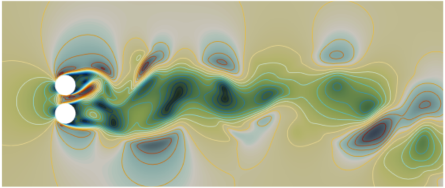


Numerical results

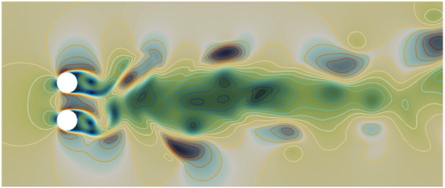




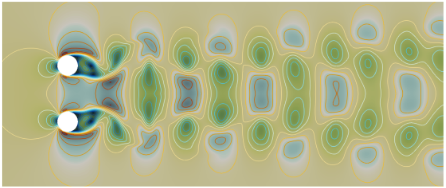
Numerical results



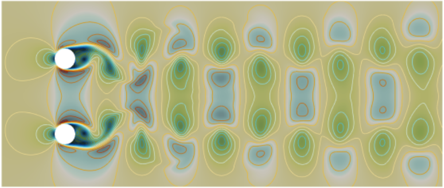
(a) $\ell/\mathcal{D} = 1.5$



(b) $\ell/\mathcal{D} = 2.0$

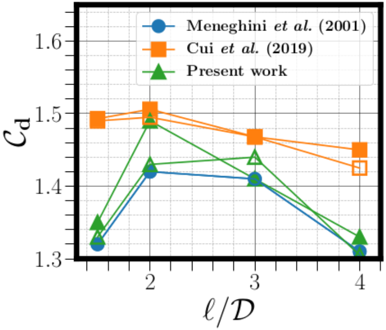


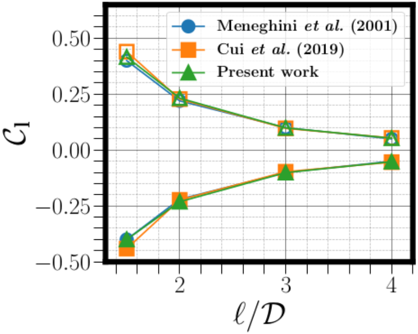
(c) $\ell/\mathcal{D} = 3.0$



(d) $\ell/\mathcal{D} = 4.0$

Numerical results





Conclusion and perspectives

Conclusion and perspectives

- ➡ The accurate conservative level set with an important improvement of the modification of the re-initialization step is used for the two-way coupling of a fluid with rigid bodies.
- ➡ The performance of this method has been proven to correctly simulate flow over two cylinder in tandem and side-by-side arrangement
  - ✔ The results of the mean drag and lift coefficients and the Strouhal number were compared with other author's results.
- ➡ offer quite promising perspectives for future applications to more complex industrial problems.

Thank you for your kind attention ☺ !
Questions ?

Acknowledgments

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References

Chiodi, R. and O. Desjardins (2017). "A reformulation of the conservative level set reinitialization equation for accurate and robust simulation of complex multiphase flows". In: *Journal of Computational Physics* 343, pp. 186–200.

Chorin, A. J. (1968). "Numerical solution of the Navier–Stokes equations". In: *Mathematics of Computation* 22.104, pp. 745–762.

Kim, J. and P. Moin (1985). "Application of a fractional-step method to incompressible Navier–Stokes equations". In: *Journal of Computational Physics* 59.2, 308–323.

Sahut, G. et al. (2020). "Evaluation of Level Set reinitialization algorithms for phase change simulation on unstructured grids". In: *La Houille Blanche* 2, pp. 43–48.